SNAP Centre Workshop

Scientific Notation

Introduction

Scientific notation is a way of writing very large or very small numbers that would otherwise be inconvenient to write using standard decimal notation. It splits a given number into two parts: the **coefficient term**, which is a decimal number with an absolute value between 1 and 10, and the **exponential term**, which is 10 raised to a positive or negative integer, and multiplies them together.

Expressing Large Numbers in Scientific Notation

Example 1 Write $1\ 560\ 000\ 000$ using scientific notation.

First, we need to determine what our coefficient term will be.

It may be tempting to select 156 *as our coefficient term since it is followed by a long string of* 0*s, but since our goal is to have a digit in the ones place – or "to the left" of the decimal point – and the rest as decimal fractions,* 156 *is not the correct choice. Instead we choose* 1.56*, which has an absolute value between* 1 *and* 10*, and gives us the following equation:*

$$1\ 560\ 000\ 000 = 1.56 \times 10^x$$

Next, we divide both sides by 1.56 *to isolate the exponential term.*

$$\frac{1\ 560\ 000\ 000}{1.56} = \frac{1.56 \times 10^x}{1.56}$$

$$1\ 000\ 000\ 000 = 10^x$$

Taking the $\log_{10}$ *of both sides allows us to solve for* x*.*

$$\log_{10} 1\ 000\ 000\ 000 = \log_{10} 10^x$$

$$9 = x$$

Substituting the value we found for x *back into our original equation gives us our number expressed in scientific notation.*

$$\mathbf{1\ 560\ 000\ 000 = 1.56 \times 10^9}$$

Example 1 serves as a comprehensive explanation of how to express a given number using scientific notation, however, it is much faster (and often quite a bit easier) to express a number using scientific notation solely by observation.

Scientific Notation by Observation

Example 2 Write $1\ 560\ 000\ 000$ using scientific notation, by observation.

*The process of choosing our coefficient term remains the same, so we know from **Example 1** that it will be* 1.56.

$$1\ 560\ 000\ 000 = 1.56 \times 10^{x}$$

Solving for x by observation rather than by algebraic manipulation involves counting the number of places the decimal in our original term would need to move in order to become our coefficient term.

The decimal point is not normally written when there are no decimal fractions, however, it is assumed to be to the right of the ones place. For this example, we will replace it and highlight the number of places it needs to move in bold.

$$1\ \mathbf{560\ 000\ 000}. = \ 1.56 \times 10^{x}$$

There are 9 places highlighted in bold, so we know our x value must be 9.

$$\mathbf{1\ 560\ 000\ 000 = 1.56 \times 10^{9}}$$

*This confirms our answer from **Example 1**.*

Expressing Very Small Numbers Using Scientific Notation

Expressing very small numbers in scientific notation involves a similar process, however, the exponent of the exponential term will be a negative number since we want to divide our coefficient term by a power of 10.

Example 3 Write 0.0000400025 using scientific notation, by observation.

*As in **Example 1,** our first step is to find our coefficient term.*

$$0.0000400025 = 4.00025 \times 10^{-x}$$

Next, we want to count how many places the decimal point would need to move in order to become our coefficient term.

$$0.\mathbf{00004}00025 = 4.00025 \times 10^{-x}$$

There are five places highlighted in bold, so our x value is 5.

$$\mathbf{0.0000400025 = 4.00025 \times 10^{-5}}$$

Addition/Subtraction

It is only possible to add or subtract numbers written using scientific notation if the exponential terms match. The exponential terms are factored out, then the addition or subtraction operation is performed on the coefficient terms.

Example 4 $6.078 \times 10^{-3} - 3.155 \times 10^{-3}$ $\quad$ $Find\ the\ difference.$

The exponential terms are both 10^{-3} *which allows us to factor them out and find the difference between coefficient factors.*

$$= (6.078 - 3.155) \times 10^{-3}$$

$$= 2.923 \times 10^{-3}$$

If exponential terms do not match, the numbers can be adjusted by factoring powers of 10 out of the exponential term so that they do match, and the resulting sum or difference remains in scientific notation.

Example 5 $7.2 \times 10^{4} + 5.201 \times 10^{6}$ $\quad$ $Find\ the\ sum.$

Our goal here is to adjust one of our two numbers so their exponential terms match.

If we adjust the second number so that its exponential term becomes 10^{4}*, the exponential terms match, however, the coefficient term is multiplied by the* 10^{2} *we factored out of the exponential term, becoming* 520.1*. As a result, our final sum – once calculated – will no longer be using scientific notation.*

If we adjust the first number so that its exponential term becomes 10^{6}*, the exponential terms match, and the coefficient term will be multiplied by the* 10^{-2} *we factored out of the exponential term, resulting in a final sum that remains in scientific notation.*

$$= (7.2 \times (10^{6})(10^{-2})) + 5.201 \times 10^{6}$$

$$= 0.072 \times 10^{6} + 5.201 \times 10^{6}$$

With our numbers properly adjusted, all we need to do is factor out the 10^{6} *and add the coefficient terms.*

$$= (0.072 + 5.201) \times 10^{6}$$

$$= \mathbf{5.273 \times 10^{6}}$$

Multiplication/Division

Multiplication and division are very straightforward operations when using numbers in scientific notation; when multiplying, the product of the coefficient terms is found, followed by the product of the exponential terms. If the new expression is no longer in scientific notation after multiplication, it is adjusted accordingly.

Example 6 $(3.4 \times 10^4)(5.2 \times 10^{-6})$ $\quad$ $Find\ the\ product.$

Our first step is organizing our terms, grouping coefficient terms and exponential terms together.

$$= [(3.4)(5.2)] \times [(10^4)(10^{-6})]$$

Once grouped, we can find the products of the coefficient terms and exponential terms. Keep in mind that since the exponential terms share the same base, finding their product is simply a matter of adding their exponents.

$$= 17.68 \times 10^{4+(-6)}$$

$$= 17.68 \times 10^{-2}$$

Next, we adjust the result by observation. In this case, the decimal point needs to move one place to the left, and the exponent in our exponential term increases by 1.

$$= 1.768 \times 10^{-1}$$

Finally, we need to take care of significant figures. Given the terms we were originally provided, there should only be two significant figures in the final product.

$$= 1.8 \times 10^{-1}$$

Example 7 $\dfrac{3.026\times10^{-3}}{2.7\times10^{-7}}$ $\quad$ $Find\ the\ quotient.$

Like ***Example 6****, we split the problem, grouping our coefficient terms and exponential terms.*

$$= \left(\frac{3.026}{2.7}\right) \times \left(\frac{10^{-3}}{10^{-7}}\right)$$

$$= 1.12 \times 10^4$$

We did not need to adjust our coefficient term, however, we do need to adjust our significant figures.

$$= \mathbf{1.1 \times 10^4}$$

Exponents

Raising a term given in scientific notation to an exponent is similar to multiplication of terms in scientific notation. The coefficient term and exponential term are handled separately, then adjusted at the end if needed.

Example 8 $(5.36 \times 10^{-3})^2$

We can use the product rule for exponents to distribute the exponent across our terms.

$$= (5.36)^2 \times (10^{-3})^2$$

At this step, we need to make sure to multiply the exponents of our exponential term, instead of adding them.

$$= 28.73 \times 10^{-6}$$

Next, we adjust our coefficient term so that it has an absolute value between 1 and 10, and adjust the exponential term accordingly.

$$= 2.873 \times 10^{-5}$$

Finally, we adjust our significant figures.

$$= \mathbf{2.87 \times 10^{-5}}$$