

Recent Results on the Existence of a Variety of Combinatorial Designs and Tournaments

F. E. Bennett

Department of Mathematics
Mount Saint Vincent University
Halifax, Nova Scotia B3M 2J6
Canada

E-mail Address: frank.bennett@msvu.ca

Abstract

We shall present a survey of recent results on the existence of a variety of combinatorial designs and related tournaments. Briefly, a *whist tournament* $Wh(v)$ for $v = 4n$ (or $4n + 1$) players is a schedule of games (a, b, c, d) where the unordered pairs $\{a, c\}$, $\{b, d\}$ are called *partners*, the pairs $\{a, b\}$, $\{c, d\}$, $\{a, d\}$, $\{b, c\}$ are called *opponents*, such that (1) the games are arranged into $4n - 1$ (or $4n + 1$) rounds, each of n games; (2) each player plays in exactly one game in each round (or all rounds but one); (3) each player partners every other player exactly once; (4) each player opposes every other player exactly twice. A whist tournament $Wh(v)$ is said to have the *three person property* if no two games in the tournament have three common players. We briefly denote such a tournament as a $3PWh(v)$. The underlying design of a $3PWh(v)$ is a *super-simple* $(v, 4, 3)$ -BIBD, that is, a *balanced incomplete block design* of index 3 in which any two blocks of size 4 intersect in at most two points. We may think of (a, b, c, d) as the cyclic order of the four players sitting round a table. We refer to the pairs $\{a, b\}$ and $\{c, d\}$ as pairs of *opponents of the first kind*, and the pairs $\{a, d\}$ and $\{b, c\}$ as pairs of *opponents of the second kind*. We also refer to b as the *left-hand opponent* of a and as the *right-hand opponent* of c , and similar definitions apply to each of a, b, c, d . A *directedwhist tournament* $DWh(v)$ is a $Wh(v)$ in which each player is a left (resp., right) hand opponent of every other player exactly once. A *triplewhist tournament* $TWh(v)$ is a $Wh(v)$ in which each player is an opponent of the first (resp., second) kind exactly once with every other player. Whist tournaments which are simultaneously both triplewhist and directedwhist are called *directed triplewhist tournaments* and denoted briefly by $DTWh(v)$. Ever since the problem of existence of whist tournaments $Wh(v)$ was completely settled in 1975, the focus has turned to whist tournaments with additional properties. Such special whist tournaments include at least directedwhist tournaments, triplewhist tournaments, whist tournaments with the three person property, and Z -cyclic whist tournaments. As more and more results have been obtained, the attention has turned to whist tournaments that satisfy more than one of the above mentioned criteria simultaneously. We survey the existence results for such a variety of whist tournaments and mention some open problems.